Making and Using Graphs

After studying this chapter you will be able to:

◆ Make and interpret a scatter diagram, a time-series graph and a cross-section graph

◆ Distinguish between linear and non-linear relationships and between relationships that have a maximum and a minimum

◆ Define and calculate the slope of a line

◆ Graph relationships among more than two variables

Three Kinds of Lie

Benjamin Disraeli, a British prime minister in the late nineteenth century, is reputed to have said that there are three kinds of lie: lies, damned lies and statistics. One of the most powerful ways of conveying statistical information is in the form of a graph. Like statistics, graphs can lie. But the right graph does not lie. It reveals a relationship that would otherwise be obscure. ◆ Graphs were invented in the eighteenth century, but with the development of the mass media and personal computers, graphs have become as important as words and numbers. What do graphs reveal and what can they hide? How and why do economists use graphs? How are graphs used to present economic information? ◆ It is often said that in economics, everything depends on everything else. Changes in the quantity of wheat for world consumption are caused by changes in the world price of wheat, the temperature, unusual weather conditions and many other factors. Graphs can help us interpret relationships among all these factors because a picture tells a thousand stories.

◆ ◆ ◆ ◆ In this chapter, you are going to look at the kinds of graph that are used in economics. You are going to learn how to make them and read them. You are also going to learn how to calculate the strength of the effect of one variable on another. If you are already familiar with graphs, you may want to skip (or skim) this chapter. But before you do, try the last few problems at the end of the chapter to check your understanding. It may not be as good as you think.

Graphing Data

Graphs represent a quantity as a distance on a line. Figure 2.1 shows two different quantities on one graph. Temperature is shown on the horizontal line. The quantity is measured as the distance on a scale, in degrees centigrade. Movements from left to right show increases in temperature. Movements from right to left show decreases in temperature. The point marked 0 represents 0 degrees or freezing point. To the right of zero, the temperatures are positive. To the left of zero, the temperatures are negative (as indicated by the minus sign in front of the numbers). Height is measured on the vertical line in thousands of metres from sea level. The point marked 0 represents sea level. Points above zero represent height above sea level. Points below zero (indicated by a minus sign) represent depth below sea level. There are no rigid rules about the scale for a graph. The scale is determined by the range of the variable being graphed and the space available for the graph.
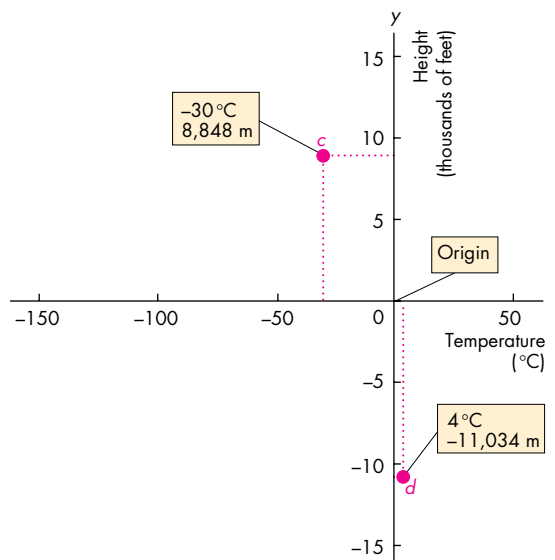
Two-variable Graphs

The two scale lines in Figure 2.1 are called *axes*. The vertical line is called the *y*-axis and the horizontal line is called the *x*-axis. The letters *x* and *y* appear as labels on the axes of Figure 2.1. Each axis has a zero point shared by the two axes. The zero point, common to both axes, is called the *origin*.

To show something in a two-variable graph, we need two pieces of information. For example, Mount Everest, the world's highest mountain, is 8,848 metres high and, on a particular day, the temperature at its peak is −30 °C. We show this information in Figure 2.1 by marking the height of the mountain on the *y*-axis at 8,848 metres and the temperature on the *x*-axis at −30 °C. The values of the two variables that appear on the axes are marked by point *c*. The values which represent the depth and temperature of the world's deepest oceanic trench are marked by point *d*.

Two lines, called *coordinates*, can be drawn from points *c* and *d*. The line running from *c* to the horizontal axis is the *y*-coordinate, because its length is the same as the value marked off on the *y*-axis. Similarly, the line running from *d* to the vertical axis is the *x*-coordinate, because its length is the same as the value marked off on the *x*-axis.

Figure 2.1   Graphing Two Variables

The relationship between two variables is graphed by drawing two axes perpendicular to each other. Height is measured here on the *y*-axis. Point *c* represents the top of Mount Everest, 8,848 metres above sea level (measured on the *y*-axis) with a temperature of −30 °C (measured on the *x*-axis). Point *d* represents the depth of the Mariana Trench in the Pacific Ocean, 11,034 metres below sea level with a temperature of 4 °C.

Economists use graphs like the one in Figure 2.1 to reveal and describe the relationships among economic variables because one picture can tell many stories. The main types of graph used in economics are:

◆   Scatter diagrams.
◆   Time-series graphs.
◆   Cross-section graphs.

Let's look at each of these types of graph and the stories they can tell.

Scatter Diagrams

A **scatter diagram** plots the value of one economic variable against the value of another variable. Such a graph is used to reveal whether a relationship

Figure 2.2 A Scatter Diagram ◆
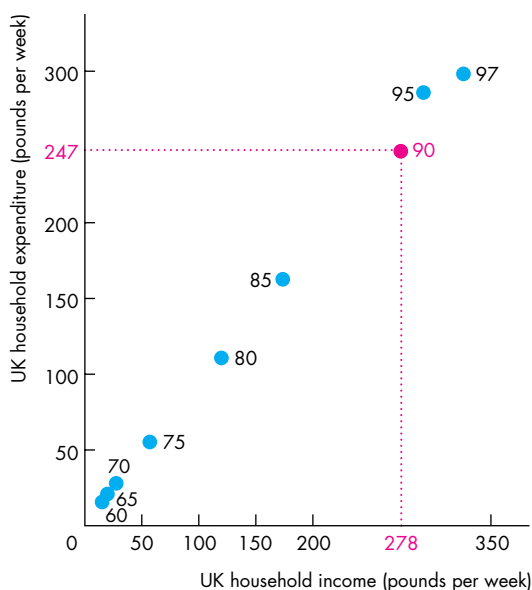
A scatter diagram shows the relationship between two variables. This scatter diagram shows the relationship between average weekly household expenditure and average weekly household income during the years 1960 to 1997. Each point shows the values of the two variables in a specific year; the year is identified by the two-digit number. For example, in 1990 average household expenditure was £247 per week and average income was £278 per week. The pattern formed by the points shows that as UK household income increases, so does household expenditure.

exists between two economic variables. It is also used to describe a relationship.

The Relationship Between Expenditure and Income   Figure 2.2 shows a scatter diagram of the relationship between consumer expenditure and income. The x-axis measures household income and the y-axis measures household expenditure. Each point shows average household expenditure and income in the United Kingdom in a given year between 1960 and 1997. The points for all seven years are 'scattered' within the graph. Each point is labelled with a two-digit number that shows us its year. For example, the point marked 90 shows us that in 1990, each household spent £247 a week on average and had an income of £278 a week.

This graph shows us that a relationship exists between household income and expenditure. The dots form a pattern which shows us that when income increases, expenditure also increases on average.

Other Relationships   Figure 2.3 shows two other scatter diagrams. Part (a) shows the relationship between the price of cigarettes and the percentage of the population over 15 years old who smoke. The pattern formed by the points shows us that as the price of cigarettes has risen, the proportion of the population who smoke has fallen.

Part (b) looks at inflation and unemployment in the European Union. The pattern formed by the points in this graph does not reveal a clear relationship between the two variables. The graph shows us, by its lack of a distinct pattern, that there is no clear relationship between inflation and unemployment.
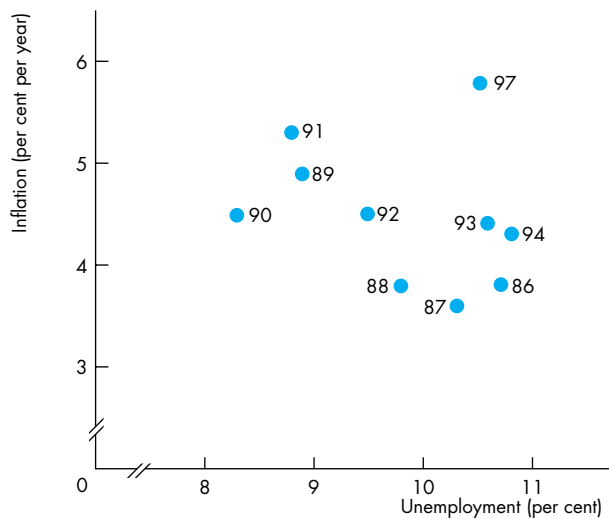
Correlation and Causation   A scatter diagram that shows a clear relationship between two variables, such as Figure 2.2 or Figure 2.3(a), tells us that the two variables are highly correlated. When a high correlation is present, we can predict the value of one variable from the value of the other. But correlation does not imply causation. Of course, it is likely that high income causes high spending, and rising cigarette prices cause a reduction in the percentage of people who smoke, but sometimes a high correlation arises by coincidence or the effect of a third variable.

Breaks in the Axes   Three of the axes in Figure 2.3 have breaks in them, as shown by the small gaps. The breaks indicate that there are jumps from the origin, 0, to the first values recorded. For example, the break is used on the x axis in part (a) because in the period covered by the graph, the proportion who smoke never falls below 30 per cent. With no break, there would be a lot of empty space. All the points would be crowded into the right-hand side, and we would not be able to see clearly whether a relationship existed between these two variables. By breaking the axes we are able to bring the relationship into view. In effect, we use a zoom lens to bring the relationship into the centre of the graph and magnify it so that it fills the graph.

A scatter diagram enables us to see the relationship between two economic variables. But it

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Figure 2.3 More Scatter Diagrams
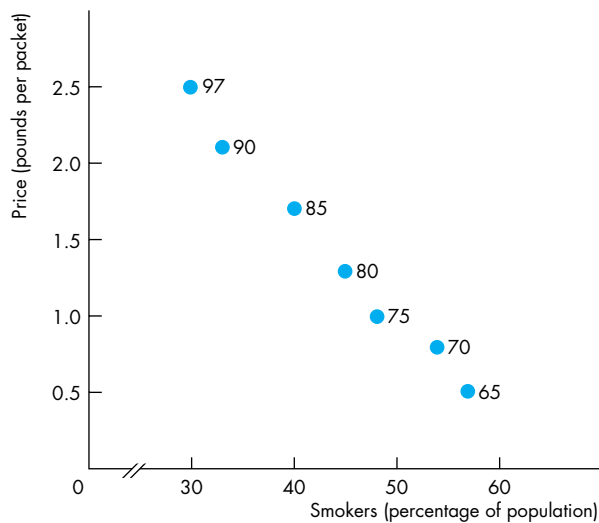
(a) Price and smoking in the United Kingdom

(b) EU unemployment and inflation

Part (a) is a scatter diagram that plots the price of a packet of cigarettes against the percentage of the population who are smokers for years between 1965 and 1997. This graph shows that as the price of cigarettes has risen, the percentage of people who smoke has decreased. Part (b) is

a scatter diagram that plots the inflation rate against the unemployment rate for the European Union as a whole. This graph shows that inflation and unemployment are not closely related.

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does not give us a clear picture of how these variables evolve over time. To see the evolution of economic variables, we use a time-series graph.

Time-series Graphs

A **time-series graph** is used to show how economic variables change over time (for example, months or years). Figure 2.4 is an example of a time-series graph. Time is measured in years on the x-axis. The economic variable that we are interested in – the UK inflation rate (the percentage change in retail prices) – is measured on the y-axis. This time-series graph tells many stories quickly and easily:

1 It shows us the level of the inflation rate. When the line is a long way from the x-axis, the inflation rate is high. When the line is close to the x-axis, the inflation rate is low.

2 It shows us how the inflation rate changes, whether it rises or falls. When the line slopes

upward, as in the early 1970s, the inflation rate is rising. When the line slopes downward, as in the early 1980s, the inflation rate is falling.

3 It shows us the speed with which the inflation rate is changing. If the line rises or falls steeply, then the inflation rate is changing quickly. If the line is shallow, the inflation rate is rising or falling slowly. For example, inflation increased quickly from 1971 to 1974 but increased more slowly in the late 1980s. Similarly, inflation was generally decreasing rapidly between 1975 and 1978, but fell more slowly after 1990.

A time-series graph also reveals trends. A **trend** is a general tendency for a variable to rise or fall. You can see that inflation had a general tendency to increase from 1967 to 1975, and then a general tendency to fall towards 1997. There is also a regular tendency for smaller fluctuations within the trend periods.

A time-series graph also lets us compare different periods quickly. Figure 2.4 shows that the 1950s

Figure 2.4   A Time-series Graph
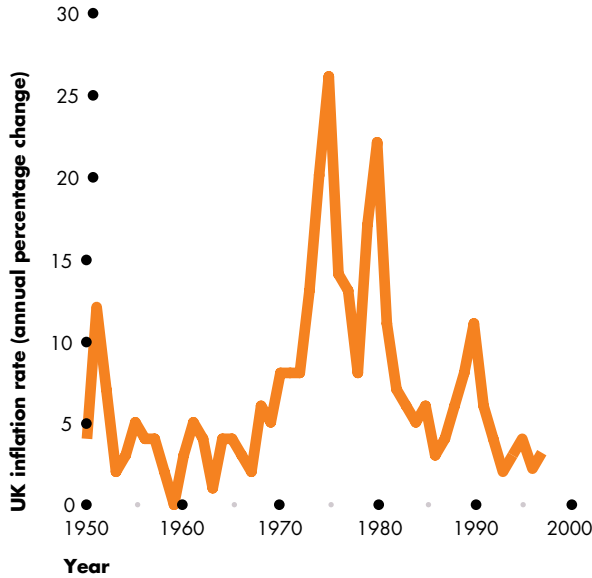
A time-series graph plots the level of a variable on the *y*-axis against time (day, week, month or year) on the *x*-axis. This graph shows the UK inflation rate each year from 1950 to 1997.

and 1960s were different from later periods because the trend in inflation was constant, neither rising nor falling.

Comparing Two Time-series   Sometimes we want to use a time-series graph to compare two different variables. For example, suppose you wanted to know how the unemployment rate fluctuates with the balance of the government's budget in the United Kingdom. You can examine the unemployment and budget balance by drawing a graph of each of them on the same time scale. We can measure the government's budget balance either as a deficit or as a surplus. Figure 2.5(a) plots the unemployment rate as the orange line and the budget balance as a surplus – the blue line. The unemployment scale is on the left side of the figure and the surplus scale is on the right side of the figure. It is not easy to see the relationship between inflation and the budget balance in Figure 2.5(a). In these situations it is often revealing to flip the scale of one of the variables over and graph it upside

Figure 2.5   Seeing Relationships in Time-series Graphs
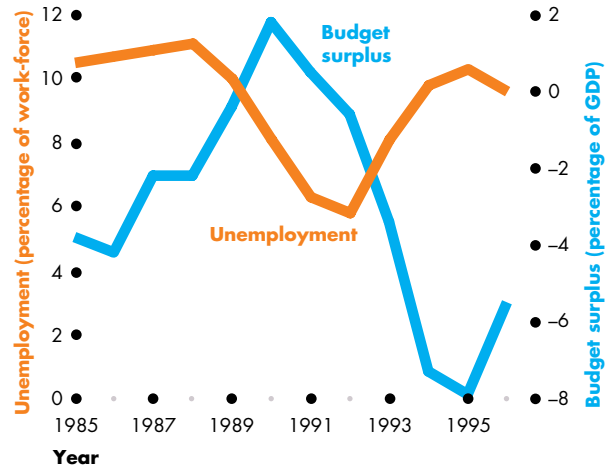
(a) Unemployment and budget surplus
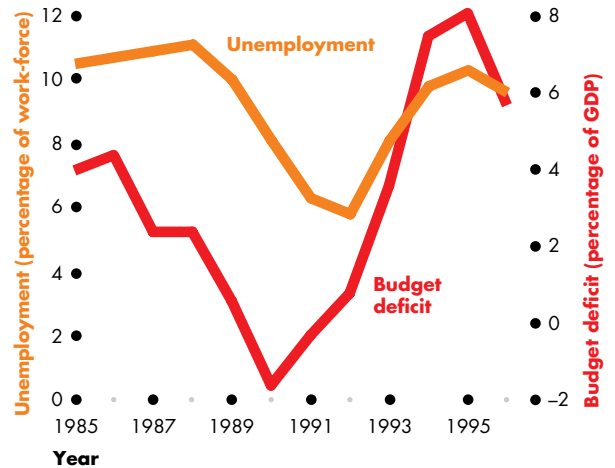
(b) Unemployment and budget deficit

These two graphs show UK unemployment and the UK government's budget balance between 1985 and 1996. The unemployment line is identical in the two parts. Part (a) shows unemployment on the left-hand scale and the budget balance as a surplus – measured on the right-hand scale. It is hard to see a relationship between inflation and unemployment. Part (b) shows unemployment again, but the government budget is measured as a deficit on the right-hand side – the scale has been inverted. The graph now reveals a tendency for unemployment and the budget deficit to move together.

Figure 2.6 A Cross-section Graph
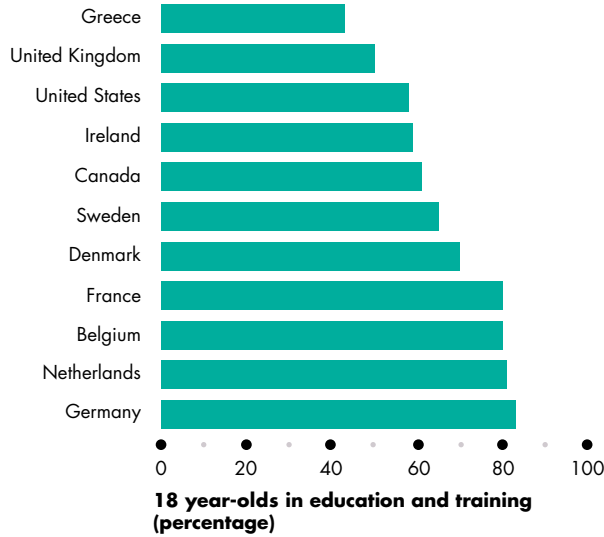
A cross-section graph shows the level of a variable across the members of a population. This graph shows the percentage of 18 year-olds in education and training in different developed countries.

down. In Figure 2.5(b) the budget balance has been inverted and plotted as a deficit. You can now see the tendency for unemployment and the budget deficit to move together.

Cross-section Graphs

A **cross-section graph** shows the values of an economic variable for different groups in a population at a point in time. Figure 2.6 is an example of a cross-section graph. It shows the percentage of 18 year-olds in education and training across different developed countries in 1996. This graph uses bars rather than dots and lines, and the length of each bar indicates the percentage. Figure 2.6 enables you to compare the level of education and training for 18 year-olds in these 11 countries much more quickly and clearly than by looking at a table of numbers.

Misleading Graphs

All types of graph – time-series graphs, scatter diagrams and cross-section graphs – can mislead. A cross-section graph gives a good example. Figure 2.7

Figure 2.7 A Misleading Graph
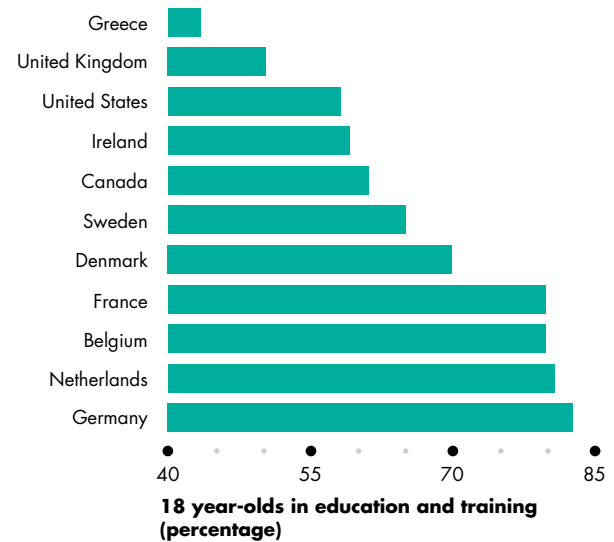
A graph can mislead when it distorts the scale on one of its axes. Here, the scale measuring the percentage of 18 year-olds in education and training has been stretched by chopping off values below 40 per cent. The result is that the comparison of percentages across the different countries is distorted. The percentage of 18 year-olds who are in education and training looks much lower in Greece and the United Kingdom when compared with countries with the highest rate.

dramatizes a point of view rather than revealing the facts. A quick glance at this graph gives the impression that the level of education and training for 18 year-olds is extremely low in Greece and the United Kingdom compared with France, Belgium, Netherlands and Germany. But a closer look reveals that the scale on the axis has been stretched and percentages between zero and 40 have been chopped off the graph. Breaks in axes, like those in Figure 2.3, are a common way of stretching and compressing axes to make a graph tell a misleading story. You should always look at the numbers on the axes before looking at the main graph to avoid being misled, even if the intention of the graph is not to mislead you. Try this with graphs in newspapers and magazines to see if you can spot a misleading graph.

We have seen how graphs are used to describe economic data and relationships between variables.

Figure 2.8 Positive (Direct) Relationships

(a) Positive linear relationship

(b) Positive becoming steeper

(c) Positive becoming less steep
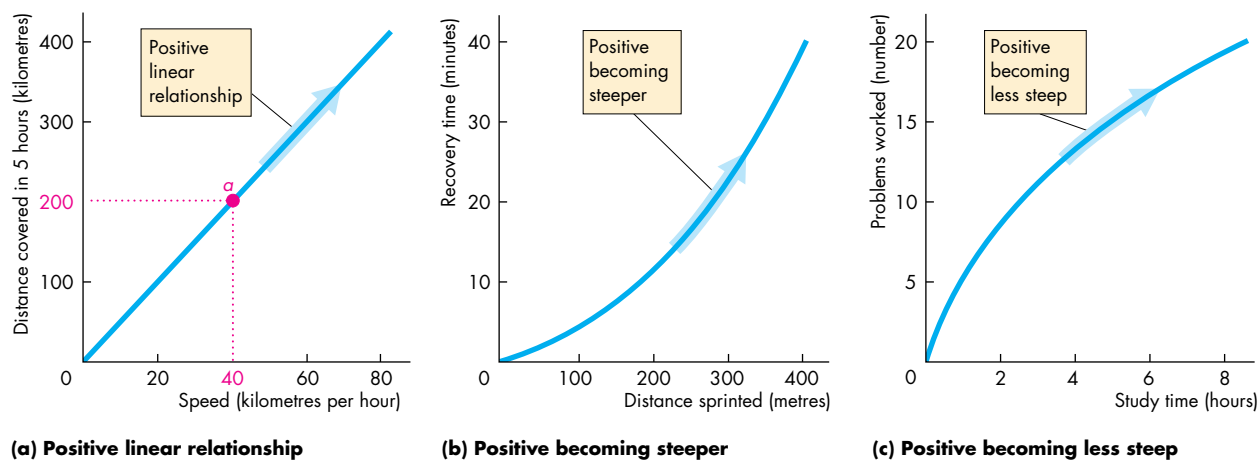
Each part of this figure shows a positive relationship between two variables. That is, as the value of the variable measured on the *x*-axis increases, so does the value of the variable measured on the *y*-axis. Part (a) shows a linear relationship – a relationship whose slope is constant as we move along the curve. Part (b) shows a positive relationship whose slope becomes steeper as we move along the curve away from the origin. It is a positive relationship with an increasing slope. Part (c) shows a positive relationship whose slope becomes flatter as we move away from the origin. It is a positive relationship with a decreasing slope.

Now we are going to see how economists use graphs in a more abstract way. Diagrams and graphs are often used by economists to explain economic models.

Graphs Used in Economic Models

The graphs used in economics are not always designed to show data. Graphs are often used to show the relationships among the variables in an economic model. Economic models are simplified descriptions of how economies, markets, firms and individuals behave. You will be learning about some of these models in the chapters that follow. Although you will encounter many different kinds of graph in economic models, there are many similarities in their pattern. Once you have learned to recognize these patterns, they will instantly convey to you the meaning of a graph. The patterns to look for are:

◆ Variables that move in the same direction.

◆ Variables that move in the opposite direction.

◆ Variables that are unrelated.

◆ Variables that have a maximum or a minimum.

Let's look at these four cases.

Variables That Move in the Same Direction

Figure 2.8 shows graphs of the relationships between two variables that move in the same direction. This is called a **positive** or a **direct relationship**. Such a relationship is shown by a line that slopes upward. In the figure, there are three types of positive relationships, one shown by a straight line and two by curved lines. A relationship shown by a straight line is called a **linear relationship**. But all the lines in these three graphs are called curves. Any line on a graph – no matter whether it is straight or curved – is called a curve.

Figure 2.8(a) shows a linear relationship between the number of kilometres travelled in 5 hours and speed. A linear relationship has a constant slope. For example, point *a* shows us that we will travel 200 kilometres in 5 hours if our speed is 40 kilometres

Figure 2.9   Negative (Inverse) Relationships

(a) Negative linear relationship   **(b) Negative becoming less steep**   **(c) Negative becoming steeper**
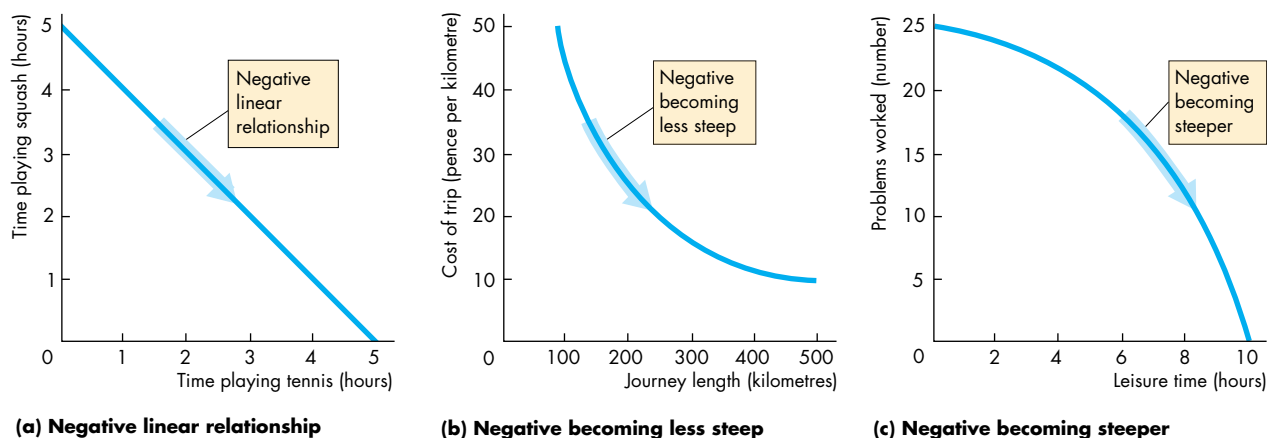
Each part of this figure shows a negative relationship between two variables. Part (a) shows a linear relationship – a relationship whose slope is constant as we travel along the curve. Part (b) shows a negative relationship with a slope that becomes flatter as the journey length increases. Part (c) shows a negative relationship with a slope that becomes steeper as the leisure time increases.

an hour. If we double our speed to 80 kilometres an hour, we will travel 400 kilometres in 5 hours.

Part (b) shows the relationship between distance sprinted and recovery time (recovery time being measured as the time it takes the heart rate to return to normal). This relationship is upward-sloping but the slope changes. The curved line starts out with a gentle slope but then becomes steeper as we move along the curve away from the origin.

Part (c) shows the relationship between the number of problems worked by a student and the amount of study time. This relationship is also upward-sloping and the slope changes. This time the curved line starts out with a steep slope but then becomes more gentle as we move away from the origin.

Variables That Move in the Opposite Direction

Figure 2.9 shows relationships between variables that move in opposite directions. This relationship is called a **negative** or an **inverse relationship**.

Part (a) shows the relationship between the number of hours available for playing squash and the number of hours for playing tennis. One extra hour spent playing tennis means one hour less playing squash and vice versa. This relationship is negative and linear.

Part (b) shows the relationship between the cost per kilometre travelled and the length of a journey. The longer the journey, the lower is the cost per kilometre. But as the journey length increases, the cost per kilometre decreases and the fall in the cost is smaller, the longer the journey. This feature of the relationship is shown by the fact that the curve slopes downward, starting out steep at a short journey length and then becoming flatter as the journey length increases.

Part (c) shows the relationship between the amount of leisure time and the number of problems worked by a student. Increasing leisure time produces an increasingly large reduction in the number of problems worked. This relationship is a negative one that starts out with a gentle slope at a small number of leisure hours and becomes steeper as the number of leisure hours increases.

Variables That are Unrelated

There are many situations in which one variable is unrelated to another. No matter what happens to the value of one variable, the other variable remains

Figure 2.10 Variables that are Unrelated

(a) Unrelated: _y_ constant

(b) Unrelated: _x_ constant
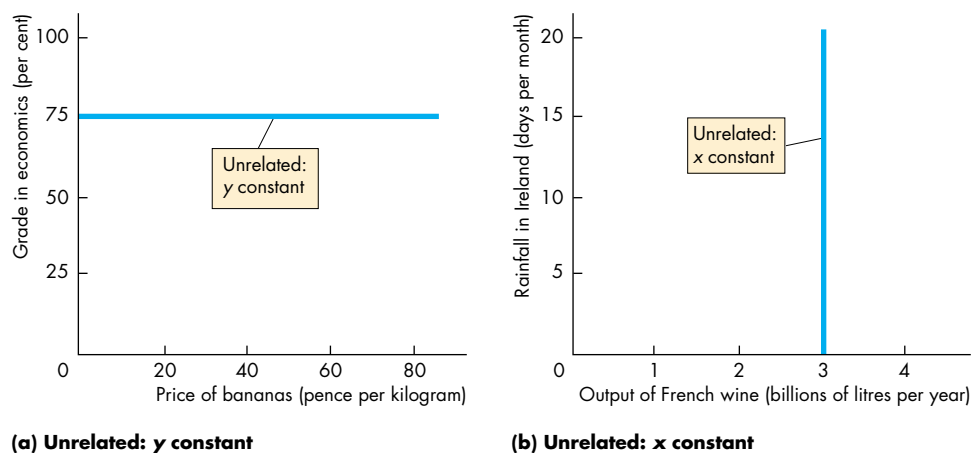
This figure shows how we can graph two variables that are unrelated to each other. In part (a), a student's grade in economics is plotted at 75 per cent regardless of the price of bananas on the _x_-axis. The curve is horizontal. In part (b), the output of the vineyards of France does not vary with the rainfall in Ireland. The curve is vertical.

constant. Sometimes we want to show the independence between two variables in a graph. Figure 2.10 shows two ways of achieving this. In Figure 2.10(a), your grade in economics is shown on the _y_-axis against the price of bananas on the _x_-axis. Your grade (75 per cent in this example) is unrelated to the price of bananas. The relationship between these two variables is shown by a horizontal straight line. This line slopes neither upward nor downward. In part (b), the output of French wine is shown on the _x_-axis and the number of rainy days a month in Ireland is shown on the _y_-axis. Again, the output of French wine (3 billion litres a year in this example) is unrelated to the number of rainy days in Ireland. The relationship between these two variables is shown by a vertical straight line.

### Variables That Have a Maximum and a Minimum

Many relationships in economic models have a maximum or a minimum. For example, firms try to make the maximum possible profit and to produce at the lowest possible cost. Figure 2.11 shows relationships that have a maximum or a minimum.

Part (a) shows the relationship between rainfall and wheat yield. When there is no rainfall, wheat will not grow, so the yield is zero. As the rainfall increases up to 10 days a month, the wheat yield

also increases. With 10 rainy days each month, the wheat yield reaches its maximum at 40 tonnes a hectare (point _a_). Rain in excess of 10 days a month starts to lower the yield of wheat. If every day is rainy, the wheat suffers from a lack of sunshine and the yield falls back almost to zero. This relationship is one that starts out with a positive slope, reaches a maximum at which its slope is zero, and then moves into a range in which its slope is negative.

Part (b) shows the reverse case – a relationship that begins with a negative slope, falls to a minimum, and then becomes positive. An example of such a relationship is the petrol cost per kilometre as the speed of travel varies. At low speeds, the car is creeping along in a traffic jam. The number of kilometres per litre is low so the petrol cost per kilometre is high. At high speeds the car is travelling faster than its most efficient speed and, again, the number of kilometres per litre is low and the petrol cost per kilometre is high. At a speed of 55 kilometres per hour, the petrol cost per kilometre travelled is at its minimum (point _b_). This relationship is one that starts out with a negative slope, reaches a minimum at which its slope is zero, and then moves into a range in which its slope is positive.

Figures 2.8 through to 2.12 show 10 different shapes of graphs that we will encounter in economic models. In describing these graphs, we

Figure 2.11 Maximum and Minimum Points

(a) Relationship with a maximum

(b) Relationship with a minimum
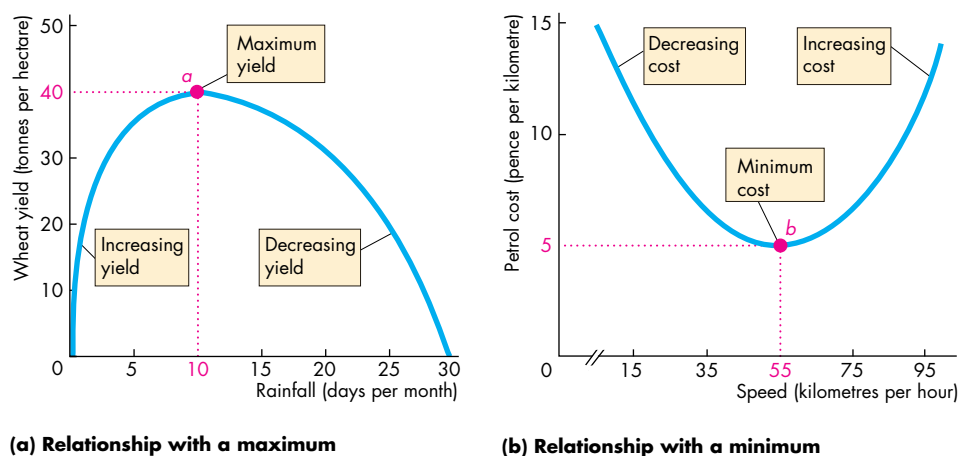
Part (a) shows a relationship that has a maximum point, *a*. The curve has a positive slope as it rises to its maximum point, is flat at its maximum, and then has a negative slope. Part (b) shows a relationship with a minimum point, *b*. The curve has a negative slope as it falls to its minimum, is flat at its minimum, and then has a positive slope.

have talked about the slopes of curves. The concept of a slope is an intuitive one. But it is also a precise technical concept. Let's look more closely at the concept of slope.

The Slope of a Relationship

The **slope** of a relationship is the change in the value of the variable measured on the y-axis divided by the change in the value of the variable measured on the x-axis. We use the Greek letter Δ to represent 'change in'. Thus Δy means the change in the value of the variable measured on the y-axis, and Δx means the change in the value of the variable measured on the x-axis. Therefore, the slope of the relationship is:

$$\frac{\Delta y}{\Delta x}$$

If a large change in the variable measured on the y-axis (Δy) is associated with a small change in the variable measured on the x-axis (Δx), the slope is large and the curve is steep. If a small change in the variable measured on the y-axis (Δy) is associated with a large change in the variable measured on the x-axis (Δx), the slope is small and the curve is flat.

We can make the idea of slope sharper by doing some calculations.

The Slope of a Straight Line

The slope of a straight line is the same regardless of where on the line you calculate it. Thus the slope of a straight line is constant. Let's calculate the slopes of the lines in Figure 2.12. In part (a), when x increases from 2 to 6, y increases from 3 to 6. The change in x is plus 4, that is, Δx is 4. The change in y is plus 3, that is, Δy is 3. The slope of that line is:

$$\frac{\Delta y}{\Delta x} = \frac{3}{4}$$

In part (b), when x increases from 2 to 6, y decreases from 6 to 3. The change in y is minus 3, that is, Δy is −3. The change in x is plus 4, that is, Δx is 4. The slope of the curve is:

$$\frac{\Delta y}{\Delta x} = \frac{-3}{4}$$

Notice that the two slopes have the same magnitude (3/4), but the slope of the line in part (a) is positive (+3/+4) = 3/4), while that in part (b) is negative (−3/+4 = −3/4). The slope of a positive relationship is positive; the slope of a negative relationship is negative.

The Slope of a Curved Line

The slope of a curved line is not constant. It depends on where on the line we calculate it. There are two ways to calculate the slope of a curved line:

Figure 2.12   The Slope of a Straight Line

(a) Positive slope

(b) Negative slope
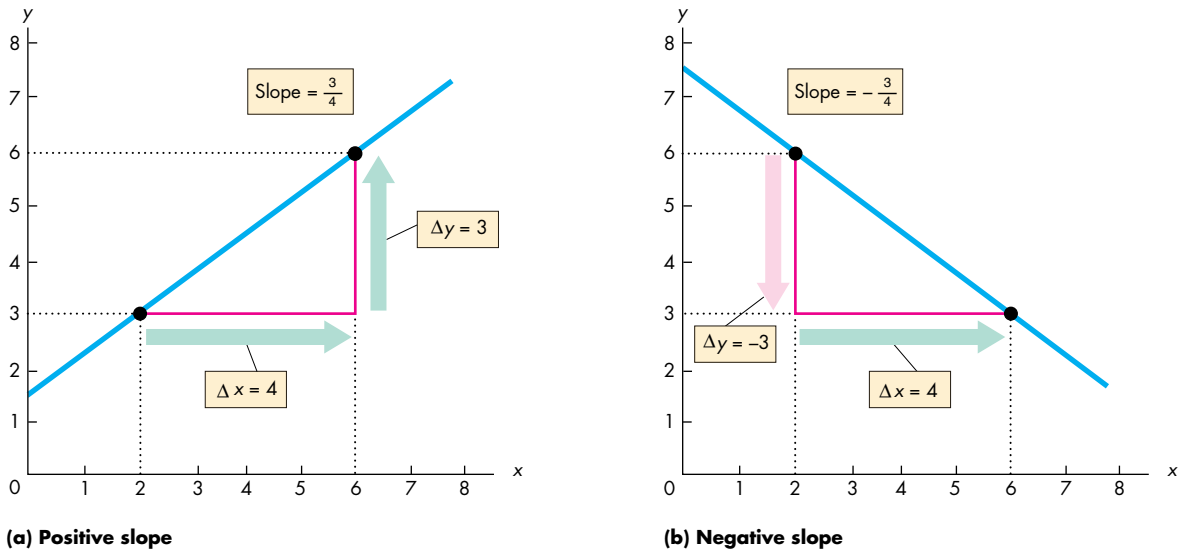
To calculate the slope of a straight line, we divide the change in the value of the variable measured on the *y*-axis (Δ*y*) by the change in the value of the variable measured on the *x*-axis (Δ*x*). Part (a) shows the calculation of a positive slope. When *x* increases from 2 to 6, Δ*x* equals 4. That change in *x* brings about an increase in *y* from 3 to 6,

so Δ*y* equals 3. The slope (Δ*y*/Δ*x*) equals 3/4. Part (b) shows the calculation of a negative slope. When *x* increases from 2 to 6, Δ*x* equals 4. That increase in *x* brings about a decrease in *y* from 6 to 3, so Δ*y* equals –3. The slope (Δ*y*/Δ*x*) equals –3/4.

at a point on the curve or across an arc of the curve. Let's look at them.

Slope at a Point   To calculate the slope at a point on a curve, you need to construct a straight line that has the same slope as the curve at the point in question. Figure 2.13 shows how this is done. Suppose you want to calculate the slope of the curve at point a. Place a ruler on the graph so that it touches point a and no other point on the curve, then draw a straight line along the edge of the ruler. The straight red line in part (a) is this line and it is the tangent to the curve at point a. If the ruler touches the curve only at point a, then the slope of the curve at point a must be the same as the slope of the edge of the ruler. If the curve and the ruler do not have the same slope, the line along the edge of the ruler will cut the curve instead of just touching it.

Having found a straight line with the same slope as the curve at point a, you can calculate the slope of the curve at point a by calculating the slope

of the straight line. Along the straight line, as x increases from 0 to 4 ($\Delta x = 4$) y increases from 2 to 5 ($\Delta y = 3$). Therefore, the slope of the line is:

$$\frac{\Delta y}{\Delta x} = \frac{-3}{4}$$

Thus the slope of the curve at point a is 3/4.

Slope Across an Arc   Calculating a slope across an arc is similar to calculating an average slope. An arc of a curve is a piece of a curve. In Figure 2.14(b), we are looking at the same curve as in part (a), but instead of calculating the slope at point a, we calculate the slope across the arc from b to c. Moving along the arc from b to c, x increases from 3 to 5 and y increases from 4 to 5.5. The change in x is 2 ($\Delta x = 2$) and the change in y is 1.5 ($\Delta y = 1.5$). Therefore, the slope of the line is:

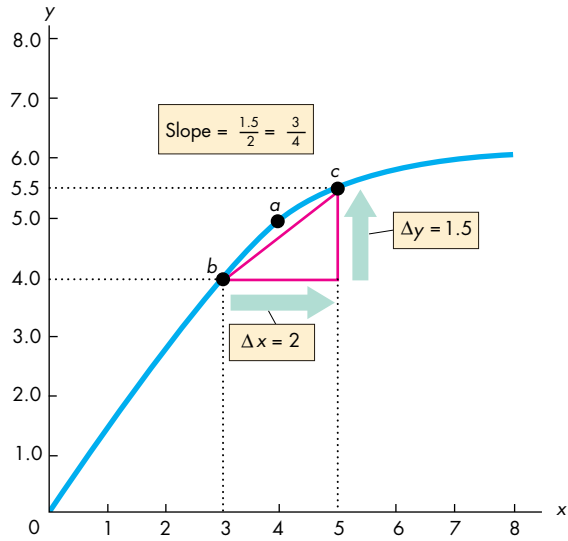$$\frac{\Delta y}{\Delta x} = \frac{1.5}{2} = \frac{3}{4}$$

Thus the slope of the curve across the arc bc is 3/4.

Figure 2.13 The Slope of a Curve

(a) Slope at a point

(b) Slope across an arc

To calculate the slope of the curve at point *a*, draw the red line that just touches the curve at *a* – the tangent. The slope of this straight line is calculated by dividing the change in *y* by the change in *x* along the line. When *x* increases from 0 to 4, Δx equals 4. That change in *x* is associated with an increase in *y* from 2 to 5, so Δy equals 3. The slope of the red line is 3/4. So the slope of the curve at point *a* is 3/4.

To calculate the average slope of the curve along the arc *bc*, draw a straight line from *b* to *c* in part (b). The slope of the line *bc* is calculated by dividing the change in *y* by the change in *x*. In moving from *b* to *c*, Δx equals 2, and Δy equals 1.5. The slope of the line *bc* is 1.5 divided by 2, or 3/4. So the slope of the curve across the arc *bc* is 3/4.

In this particular example, the slope of the arc *bc* is identical to the slope of the curve at point *a* in part (a). But the calculation of the slope of a curve does not always work out so neatly. You might have some fun constructing counter-examples. If you visit the Parkin, Powell and Matthews Web site you can find out how calculus is used to measure the slope of an arc in an algebraic example.

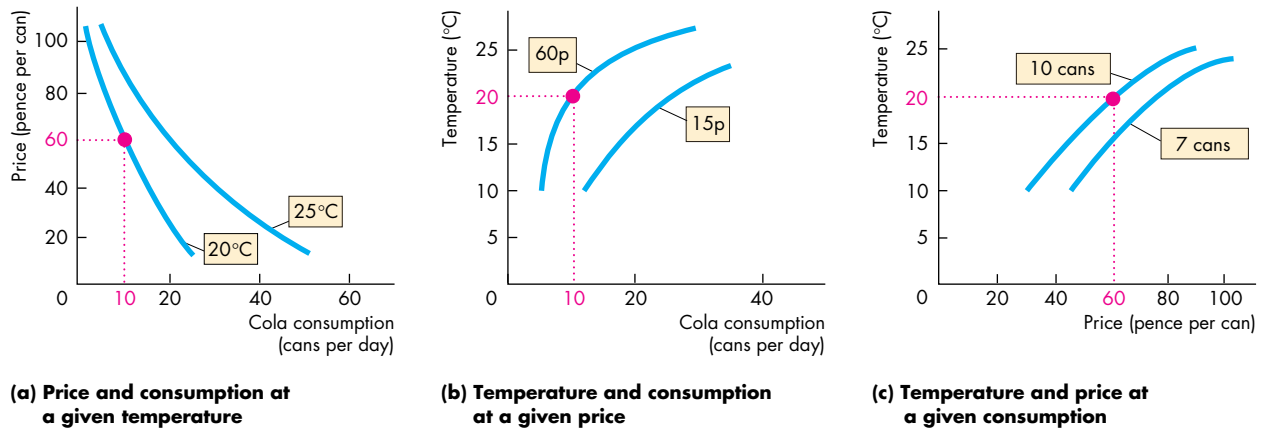## Graphing Relationships Among More Than Two Variables

We have seen that we can graph a single variable as a point on a straight line and we can graph the relationship between two variables as a point formed by the *x* and *y* coordinates in a two-

dimensional graph. You may be thinking that although a two-dimensional graph is informative, most of the things in which you are likely to be interested involve relationships among many variables, not just two.

For example, the amount of cola consumed depends on the price of a can of cola and the temperature. If a can of cola is expensive and the temperature is low, people drink much less cola than when a can of cola is inexpensive and the temperature is high. For any given price of a can of cola, the quantity consumed varies with the temperature, and for any given temperature, the quantity of cola consumed varies with its price.

The graph in Figure 2.14 tells this more complicated story. Figure 2.14 shows the relationship among three variables. The table shows the number of cans of cola consumed each day at various temperatures and prices for cans of cola.

Figure 2.14   Graphing a Relationship Among Three Variables

(a) Price and consumption at a given temperature

(b) Temperature and consumption at a given price

(c) Temperature and price at a given consumption

Price (pence per can)	Cola consumption (cans per day)			
	10 °C	**15 °C**	**20 °C**	**25 °C**
15	12	18	25	50
30	10	12	18	37
45	7	10	13	27
60	5	7	10	20
75	3	5	7	14
90	2	3	5	10
105	1	2	3	6

The quantity of cola consumed depends on its price and the temperature. The table gives some hypothetical numbers that tell us how many cans of cola are consumed each day at different prices and different temperatures. For example, if the price is 30 pence per can and the temperature is 10 °C, 10 cans of cola are consumed. In order to graph a relationship among three variables, the value of one variable is held constant. Part (a) shows the relationship between price and consumption, holding temperature constant. One curve holds temperature at 25 °C and the other at 20 °C. Part (b) shows the relationship between temperature and consumption, holding price constant. One curve holds the price at 60 pence and the other at 15 pence. Part (c) shows the relationship between temperature and price, holding consumption constant. One curve holds consumption at 10 cans and the other at 7 cans.

To graph a relationship that involves more than two variables, we consider what happens if all but two of the variables are held constant. When we hold other things constant, we are using the *ceteris paribus* assumption that is described in Chapter 1, p. 14. An example is shown in Figure 2.14(a). There, you can see what happens to the quantity of cola consumed when the price of a can varies while the temperature is held constant. The line labelled 20 °C shows the relationship between cola consumption and the price of a can of cola when the temperature stays at 20 °C. The numbers used to plot this line are those in the third column of the table in Figure 2.14. For example, when the temperature is 20 °C, 10 cans are consumed when the price is 60 pence and 18 cans are consumed when the price is 30 pence. The curve labelled

25 °C shows the consumption of cola when the price varies and the temperature is 25 °C.

We can also show the relationship between cola consumption and temperature while holding the price of cola constant, as shown in Figure 2.14(b). The curve labelled 60 pence shows how the consumption of cola varies with the temperature when cola costs 60 pence, and a second curve shows the relationship when cola costs 15 pence. For example, at 60 pence a can, 10 cans are consumed when the temperature is 20 °C and 20 cans when the temperature is 25 °C.

Figure 2.14(c) shows the combinations of temperature and price that result in a constant consumption of cola. One curve shows the combination that results in 10 cans a day being consumed, and the other shows the combination that results in 7 cans

a day being consumed. A high price and a high temperature lead to the same consumption as a lower price and a lower temperature. For example, 10 cans are consumed at 20 °C and 60 pence per can and at 25 °C and 90 pence per can.

The three graphs in Figure 2.14 tell a story which has taken us more than 500 words to explain. Once you have learned to read graphs, you too will be able to see this much information at a glance.

With what you have learned about graphs, you can move forward with your study of economics. There are no graphs in this book that are more complicated than those that have been explained here.

Summary

Key Points

Graphing Data (pp. 24–28)

◆ A scatter diagram plots the value of one economic variable against the value of another and reveals whether or not there is a relationship between the two variables. If there is a relationship, the graph reveals its nature.

◆ A time-series graph shows the trend of economic variables over time, and the level, direction of change and speed of change of each variable.

◆ A cross-section graph shows how a variable changes across the members of a population.

◆ A graph can mislead if its scale is stretched (or squeezed) to exaggerate (or understate) a variation.

Graphs Used in Economic Models (pp. 29–32)

◆ Graphs are used to show relationships among variables in economic models.

◆ Relationships can be positive (an upward-sloping curve), negative (a downward-sloping curve), unrelated (a horizontal or vertical curve), positive and then negative (a maximum), and negative and then positive (a minimum).

The Slope of a Relationship (pp. 32–34)

◆ The slope of a relationship is calculated as the change in the value of the variable measured on the y-axis divided by the change in the value of the variable measured on the x-axis – $\Delta y/\Delta x$.

◆ A straight line has a constant slope, but a curved line has a varying slope.

◆ To calculate the slope of a curved line, we calculate the slope at a point or across an arc.

Graphing Relationships Among More Than Two Variables (pp. 34–36)

◆ To graph a relationship among more than two variables, we hold constant the values of all the variables except two.

◆ We then plot the value of one of the variables against the value of another.

Key Figures ◆

Key Terms

Review Questions

1 What are the three types of graph used to show economic data?

2 Give an example of a scatter diagram.

3 Give an example of a time-series graph.

4 Give an example of a cross-section graph.

5 List three things that a time-series graph shows quickly and easily.

6 What do we mean by trend?

7 How can a graph mislead?

8 Draw some graphs to show the relationships between two variables:

a That move in the same direction.
b That move in opposite directions.
c That have a maximum.
d That have a minimum.

9 Which of the relationships in Question 8 is a positive relationship and which is a negative relationship?

10 What is the definition of the slope of a curve?

11 What are the two ways of calculating the slope of a curved line?

12 How do we graph relationships among more than two variables?

Problems

1 The unemployment rate in the United Kingdom between 1986 and 1996 was as follows:

Year	Unemployment rate (per cent per year)
1986	11.1
1987	10.0
1988	8.1
1989	6.3
1990	5.8
1991	8.1
1992	9.8
1993	10.3
1994	9.4
1995	8.1
1996	7.4

Draw a time-series graph of these data and use your graph to answer the following questions:

a In which year was unemployment highest?
b In which year was unemployment lowest?
c In which years did unemployment increase?
d In which years did unemployment decrease?

e In which year did unemployment increase most?
f In which year did unemployment decrease most?
g What have been the main trends in unemployment?

2 The UK government's budget deficit as a percentage of GDP between 1986 and 1996 was as follows:

Year	budget deficit (percentage of GDP)
1986	2.2
1987	0.4
1988	−1.8
1989	−0.5
1990	0.6
1991	3.4
1992	7.3
1993	7.9
1994	5.5
1995	5.5
1996	5.9

38 ● **C h a p t e r 2** Making and Using Graphs

Use these data together with those in Problem 1 to draw a scatter diagram showing the relationship between unemployment and the deficit. Then use your graph to determine whether a relationship exists between unemployment and the deficit and whether it is positive or negative.

3 Use the following information on the relationship between two variables x and y.

x	0	1	2	3	4	5	6	7	8
y	0	1	4	9	16	25	36	49	64

a Draw a scatter diagram of the relationship between variables x and y.
a Is the relationship between x and y positive or negative?
b Does the slope of the relationship rise or fall as the value of x rises?

4 Using the data in Problem 3:

a Calculate the slope of the relationship between x and y when x equals 4.
b Calculate the slope of the arc when x rises from 3 to 4.
c Calculate the slope of the arc when x rises from 4 to 5.
d Calculate the slope of the arc when x rises from 3 to 5.
e What do you notice that is interesting about your answers to (b), (c) and (d), compared with your answer to (a)?

5 Calculate the slopes of the following two relationships between two variables x and y:

a
x	0	2	4	6	8	10
y	20	16	12	8	4	0

b
x	0	2	4	6	8	10
y	0	8	16	24	32	40

6 Draw a graph showing the following relationship between two variables x and y:

x	0	1	2	3	4	5	6	7	8	9
y	0	2	4	6	8	10	8	6	4	2

a Is the slope positive or negative when x is less than 5?
b Is the slope positive or negative when x is greater than 5?

c What is the slope of this relationship when x equals 5?
d Is y at a maximum or at a minimum when x equals 5?

7 Draw a graph showing the following relationship between two variables x and y:

x	0	1	2	3	4	5	6	7	8	9
y	10	8	6	4	2	0	2	4	6	8

a Is the slope positive or negative when x is less than 5?
b Is the slope positive or negative when x is greater than 5?
c What is the slope of this relationship when x equals 5?
d Is y at a maximum or at a minimum when x equals 5?

8 Plot the two variables y_1 and y_2 against variable x on one graph. The scale for variable x should be on the x axis. The scales for variables y_1 and y_2 should be on the left and right hands of the y axis as in the example of Figure 2.5.

x	y_1	y_2
1	5	45
2	10	40
3	15	35
4	20	30
5	25	25
6	30	20
7	35	15
8	40	10
9	45	5
10	50	0

a What is the slope of variable y_1?
b What is the slope of variable y_2?
c What is the value of x where the two lines representing y_1 and y_2 cross?
d What is the value of y_1 and y_2 at the point where the two lines cross?

9 At each value of x, add 5 to the corresponding value of y_2, and call this new variable y_3.

a Add your new variable y_3 to the graph you have drawn for Problem 7.
b What is the slope of your new variable y_3?
c What is the value of x where the lines representing y_3 and y_1 cross?

d What is the value of y_2 and y_3 at the point where the two lines cross?

e What is the value of x where the lines representing y_3 and y_1 cross?

f Look at your graph and describe in words the effect of adding 5 to each value of y_2?

10 The following table gives data on the price of a balloon ride, the temperature, and the number of rides taken per day.

Price (£ per person per ride)	Balloon rides (number per day at different temperatures)		
	18 °C	21 °C	25 °C
5.00	32	40	50
10.00	27	32	40
15.00	18	27	32
20.00	10	18	27

Draw graphs that show the relationship among

a The price and the number of rides taken.

b The number of rides taken and temperature, holding price constant.

c The temperature and price, holding the number of rides taken constant.

11 Use the link on the Parkin, Powell and Matthews Web site and find the Retail Price Index (RPI) for the latest 12 months.

a Make a graph of the RPI over the lastest 12 months.

b During the most recent month, is the RPI rising or falling?

c During the most recent month, is the rate of rise or fall increasing or decreasing?

12 Use the link on the Parkin, Powell and Matthews Web site and find the monthly unemployment rate for the latest 12 months.

a Graph the unemployment rate over the lastest 12 months.

b During the most recent month, is unemployment rising or falling?

c Is the rate of rise or fall increasing or decreasing?

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